The Faustmann Model as a Special Case

Julia Touza-Montero and Mette Termansen*

Environment Department
University of York
Heslington, York YO10 5DD (UK)
E-mail: jmtm100@york.ac.uk and mt120@york.ac.uk

Abstract. The Faustmann framework has been extensively used to deal with the rotation problem in forest economics. This paper considers the rotation problem using a general bioeconomic model which allows for cyclical dynamics as optimal forest policies. Constraining this general bioeconomic model to represent a clear-cutting regime, we show that the Faustmann-Hartman result arises as a special case.

Key words. Bioeconomic model, clear-cutting, Faustmann model, jump controls.

*The authors wish to thank Charles Perrings for supervising this work and Jim Smart for many helpful suggestions. We remain responsible for any errors.

1 Introduction

The economics of forestry has been dominated for over a century by the approach due to Faustmann (1849). However, this approach deals with a very simple and restricted class of forest management practices-clear felling. Faustmann addressed himself to the question of defining the age at which an even-aged forest stand should be harvested in order to maximise the return to forestry. He correctly formulated this problem as the maximisation of the net present value of the perpetual returns from the land and it was solved by Presser (1860) and Ohlin (1921). Thus, the Faustmann-Presser-Ohlin (FPO) model[1], or simply the Faustmann model, is the first valid representation of forest management practices. This model is intuitive and analytically simple. It constitutes the framework within which many subsequent forest rotation models have developed a more general representation of the rotational problem. The following influences upon the Faustmann rotation age have been examined in the literature; environmental benefits supplied by the forest (Hartman 1976, Strang 1983 and Snyder and Bhattacharya 1990), stochastic factors (Reed 1984, Willassen 1998), variability of timber prices and planting costs with respect to time (McConnell et al. 1983 and Newman et al. 1985), forest taxation (Chang 1982, 1983, and Englin and Klan 1990) and management within a forest-wide context (Paredes and Brodie 1989, and Swallow et al. 1997).

However, the Faustmann approach is limited in that it cannot address many ecological and economic aspects of forest ecosystem management. For example spatial interactions (Hof 1993), selective logging (Montgomery and Adams 1995), ecological interactions (Wacker 1999) and characteristics of forest owners (Tahvonen 1998). This deficiency is due to the rigidity inherent in static optimization analysis. This paper argues that a bioeconomic modelling framework is better suited to dealing with these issues.

Dynamic bioeconomic models, which follow an optimal control approach, offer a more flexible mathematical framework for renewable resource management and have been widely used in fisheries economics (Clark 1990). Such models aim to identify the optimal forest biomass over time. The optimal control framework was introduced to forest management by Naslund (1969) and Schreuder (1971) to simultaneously analyse optimal rotation and thinning. Anderson (1976) concluded that when the resource stock consists of an even-aged distribution of forest stands, with the oldest being removed during each harvest interval, the optimal harvest policy follows a Faustmann model for a single stand.

[1] Excellent reviews of the Faustmann-Presser-Ohlin (FPO) model can be found in Johanson and Löfgren (1985, pp. 73-94), and Clark (1990, pp. 268-75). Bentley and Teeguarden (1965), Samuelson (1976) and Löfgren (1983) offer very interesting surveys of the historical controversy surrounding this rotational problem.

The Faustmann rotation framework focuses on the age-class structure of forest stands since stand age is a key factor in the harvesting decision. Their biomass emphasis leads bioeconomic models to neglect forest age classes, which prevents them distinguishing between young and old stands of equal volume. Moreover, since dynamic biomass models consider the problem of determining the optimal harvest while maintaining the resource biomass at an optimal level over time, they fail to deal with rotational management which generally characterizes timber activities (Johanson and Löfgren, 1985, p. 55). In spite of these critiques, models which ignore forest age classes have been used to incorporate owner-specific variables. These models focus on analysing timber supply by nonindustrial private forest owners based on a Fisherian two period utility maximization approach augmented by the dynamics of forest timber stock and the owner's financial assets (Max and Lehman 1988, Koskela 1989, and Kuuluvainen 1990). Tahvonen (1998) and Tahvonen and Salo (1999) include owner's consumption-saving decisions without neglecting the forest rotational aspect. The solution of Tahvonen and Salo's model converges to the Hartman-Faustmann cutting criteria under restrictions on individual preferences and capital markets. However, their comparative static analysis produces different results from the Faustmann analysis. The effects of changes in stumpage price and reforestation cost are ambiguous due to substitution and income effects on the owners' utility. They also analyse the implications of changes in variables such as nonforest income and initial nonforest assets, which are not included in the Faustmann framework.

This paper explains the Faustmann problem within the context of a bioeconomic model introduced by Termansen (2000). This is a flexible framework for modelling optimal forest management which allows for cyclical dynamics. There are no *a priori* assumptions about the nature of the harvest strategy, i.e. whether it is rotational or continuous, and the length of the forest rotations are permitted to change over time. We show that the Faustmann model is a special case of this general bioeconomic model. Under the simplifying assumptions inherent in the Faustmann approach (clear-cutting technology, single even-aged stand, constant economic and biological parameters) the model generates the Faustmann rule as an optimum solution. Furthermore, when nontimber benefits are included in the model the optimal rotation length is given by Hartman solution. Thus, this study sets up an analytically simple and empirically tractable bioeconomic model which is able to identify optimal harvest for a single even-aged stand. In addition, it highlights that the controversy about the use of bioeconomic models in forestry is merely a dispute about whether or not to assume clear-cutting technology.

This paper is structured as follows. Firstly, the general bioeconomic model is introduced. Secondly, cyclical dynamics are constrained to generate clear-cutting strategies for an even-aged stand. Thirdly, the Faustmann-Hartman formula is derived as the optimal cutting rule. Finally, conclusions are drawn.

2 The Basic Forest Model

Let $x = x(t)$ represent the volume of forest biomass (m^3) at time t, and $F(x)$ be the timber growth function. This annual growth function is assumed to be a strictly concave growth function, such that $F(0) = F(k) = 0$ and $k > 0$, where k is the maximum volume a given stand can accumulate.

In this model, changes in forest biomass due to harvest activities are represented by jumps in the state variable, $x(\tau_j^+) - x(\tau_j^-)$. Where $x(\tau_j^+)$ denotes the biomass just after harvest and $x(\tau_j^-)$ is the biomass just before harvest. These jump points, denoted by τ_j, occur at discrete moments within the planning period, $\tau_j \;\epsilon\; [0, T]$, where $j = 1, ..., k$. The magnitude of the jump depends on the control parameter, v_j, which represents the volume of timber taken out from the forest at each harvest instant, $x(\tau_j^+) - x(\tau_j^-) = v_j$. Between jumps the change in the timber stock is given by the growth function.

In this model the forester's objective is to maximise the net present value of the economic returns derived from forest activities while choosing an optimal number of harvests, k, the location of the harvest moments, τ_j, and the volume taken out at each point in time, v_j. The forest economic rent includes both timber and non-timber benefits. The net timber benefits are expressed as the reward associated with the jump points and therefore equal to the net present benefit from the cutting activities, $R(v_j)$. The non-timber benefits are obtained from the flow of amenity services and ecological functions over time. These benefits are assumed to be a function of forest biomass, $\pi(x(t))$.

The maximisation problem can be expressed as,

$$\max_{\tau_j, v_j, k} \int_0^T \pi(x(t)) e^{-\delta t} dt + \sum_{j=1}^{k} R(v_j) e^{-\delta \tau_j} \tag{1}$$

subject to,

$$\dot{x} = F(x(t)) \quad except \;\; at \;\; \tau_j, \;\; j = 1...k \tag{2}$$

$$x(\tau_j^+) - x(\tau_j^-) = -v_j \;\; j = 1...k \tag{3}$$

$$x(0) = x_0 \tag{4}$$

Termansen (2000) illustrates that this model generates a large range of management alternatives which have been excluded in existing forest economic literature. The paper develops a numerical solution technique to solve the problem above excluding non-timber benefits. This is a flexible framework which identifies different optimal cyclical dynamics depending on the economic and biological characteristics of the forest system.

The following section illustrates that the model described above can represent clear-cut harvesting. We will then show that the Faustmann solution arises from a constrained version of the general model.

Clear-cutting in a Bioeconomic Model

Let's restrict the general bioeconomic model above to represent exclusively clear-cutting policies for a single even-aged stand. In this model the resource manager would only be free to choose the time location of the jump points in the state variable because the cutting regime is restricted to clear-cutting (Seierstad and Sydsaeter, 1987, pp. 207-09). We assume, as in the Faustmann model, that re-planting activities follow immediately after clear-cutting. In this case, the magnitude of the jumps are constrained to be equal to the size of the state variable before the jump minus x_p, the biomass after the re-planting, $x(\tau_j^+) - x(\tau_j^-) = -x(\tau_j^-) + x_p$. The timber price, the cost of planting and the interest rate are p, c_p and δ, respectively. The financial reward received from a jump is the gross timber benefit from the cutting, $px(\tau_j^-)$, minus the cost of planting, $c_p x_p$. Following Hartman (1976) the non-timber benefits are assumed to be a function of age of the trees, $\pi(t - \tau_j^+)$. Under these conditions the optimisation problem facing the resource manager is,

$$\max_{\tau_j} \sum_{j=1}^{k} \int_{\tau_j^+}^{\tau_{j+1}^-} \pi(t - \tau_j^+) e^{-\delta t} dt + \sum_{j=1}^{k} [px(\tau_j^-) - c_p x_p] e^{-\delta \tau_j} \tag{5}$$

subject to,

$$\dot{x} = F(x(t)) \quad except \;\; at \;\; \tau_j, \quad j = 1...k \tag{6}$$

$$x(\tau_j^+) - x(\tau_j^-) = -x(\tau_j^-) + x_p \;\; at \;\; \tau_j, \quad j = 1...k \tag{7}$$

$$x(0) = x_0 \tag{8}$$

There are two state variables, timber stock and trees age. Following Seierstad and Sydsaeter (1987, chapter 3, theorem 7 and section 4) the Hamiltonian and the necessary conditions for the optimal solution of this problem are

$$H = \pi(t - \tau_j^+) e^{-\delta t} + \lambda_1(t) F(x(t)) + \lambda_2(t) \tag{9}$$

$$\dot{\lambda_1} = -\lambda_1 F'(x(t)) \tag{10}$$

$$\dot{\lambda_2} = -\pi'(t - \tau_j^+)e^{-\delta t} \tag{11}$$

At the jump points $\tau_1^*, ...\tau_k^*$,

$$\lambda_1(\tau_j^{*+}) - \lambda_1(\tau_j^{*-}) = -pe^{-\delta\tau_j^-} + \lambda_1(\tau_j^{*+}) \tag{12}$$

which implies that $pe^{-\delta\tau_j^-} = \lambda(\tau_j^{*-})$.

$$\lambda_2(\tau_j^-) = 0 \tag{13}$$

Furthermore, at the jumps points the Hamiltonian need to satisfy,

$$H(x(\tau_j^{*+}), \lambda(\tau_j^{*+}), \tau_j^{*+}) - H(x(\tau_j^{*-}), \lambda(\tau_j^{*-}), \tau_j^{*-}) = -\delta[px(\tau_j^{*-}) - c_p x_p]e^{-\delta\tau_j^*} \tag{14}$$

Using Equation (9) we obtain,

$$\begin{aligned} &\pi(0)e^{-\delta\tau_j^{*+}} + \lambda_1(\tau_j^{*+})F(x^*(\tau_j^{*+})) + \lambda_2(\tau_j^{*+}) \\ &\qquad - \pi(\tau_j^{*-} - \tau_j^+)e^{-\delta\tau_j^{*-}} - \lambda_1(\tau_j^{*-})F(x^*(\tau_j^{*-})) - \lambda_2(\tau_j^{*-}) \\ &\qquad\qquad = -\delta[px(\tau_j^{*-}) - c_p x_p]e^{-\delta\tau_j^*} \end{aligned} \tag{15}$$

Between jumps points equations (6), (10) and (11) must hold. Condition (10) indicates that the rate at which the value of a unit of stock is changing is equal to its effect on the value of the capital stock. Condition (11) states that the rate at which a marginal increment in age changes the economic returns is decreasing at the same rate as instantaneous amenity benefits are increasing. Since the magnitude of the jump and the reward function depend on the timber stock, the costate variable, $\lambda_1(t)$, is discontinuous at the jump points. As equation (12) indicates the costate variable just before the jump is equal to the net present value of the timber price. Conditions (12) and (13) characterize the optimal cutting moments, i.e. the choice of τ_j within the fixed planning interval $[0, T]$. The right hand side of equation (15) represents the opportunity costs of postponing harvest by one instant. It is given by the 'potential' benefit that could be earned if the harvest were to be taken now and the net present value profits invested in a financial asset. In order to understand the left hand side, recall that the Hamiltonian function is the sum of the instantaneous value and the value of the future value of the growth of the capital stock. So the left hand side term indicates the change in value from postponing the harvesting of the standing trees. That is, (15) establishes that the optimal rotation length balances the marginal benefit of delaying the harvest with the opportunity cost of holding the standing trees.

We now derive the Faustmann equation as the solution of this problem. Let's assume that $\pi(t - \tau_j^+) \equiv 0$. The solution of the first-order differential equation given by equation (10) can be obtained as follows,

$$\begin{aligned}\lambda_1(t) = \lambda_1(\tau_j^+)exp\left\{-\int_{\tau_j^+}^{t} F'(x)dt\right\} \\ = \lambda_1(\tau_j^+)exp\left\{-\int_{x(\tau_j^+)}^{x(t)} \frac{F'(x)}{F(x)}dx\right\} \\ = \lambda_1(\tau_j^+)exp\left\{-ln[F(x(t))]|_{\tau_j^+}^{t}\right\} = \lambda_1(\tau_j^+)\frac{F(x(\tau_j^+))}{F(x(t))} \quad (16)\end{aligned}$$

Applying Equation (12), $pe^{-\delta\tau_{j+1}^-} = \lambda_1(\tau_{j+1}^-)$, and the stock constraint at the jump points (7) the above expression yields,

$$\lambda_1(\tau_j^+) = pe^{-\delta\tau_{j+1}^-}\frac{F(x(\tau_{j+1}^-))}{F(x_p)} \quad (17)$$

Thus, the cutting decision moment given by Equation (15) can be written as [2],

$$pe^{-\delta\tau_{j+1}^-}F(x(\tau_{j+1}^-)) - pe^{-\delta\tau_j^-}F(x(\tau_j^-)) = -\delta[px(\tau_j^-) - c_px_p]e^{-\delta\tau_j} \quad (18)$$

This cutting rule gives the optimal rotation length which maximizes the economic returns from forest activities. Given the Faustmann assumptions that the economic and biological parameters are constant, the forester faces an identical problem for each harvest, i.e. all rotation periods have the same length. Thus, if we denote the rotation interval as T_i, where $i = 1....\infty$, so that $T_i = \tau_j^- - \tau_{j-1}^+$ and $T_{i+1} = \tau_{j+1}^- - \tau_j^+$; and $T_1 = T_2 = T_3...\infty$. The Faustmann rule is obtained from equation (18),

$$pF(x(T_i)) = \delta px(T_i) + \frac{\delta[px(T_i)e^{-\delta T_i} - c_px_p]}{1 - e^{-\delta T_i}} \quad (19)$$

It will be optimal to harvest the standing forest when the marginal benefits of delaying the harvest equal the opportunity costs of waiting. The left-hand side of the equation (19) represents the increase in the net value of the standing forest over a unit time interval. The first term on the right-hand side is the income that could be earned if revenue from cutting is invested at an interest rate δ; and the second term, is the interest on the 'site value', which represents the opportunity cost of the land.

If the non-timber benefits can take positive values the model represents the case studied by Hartman (1976). We assumed, based on the Hartman model, that newly regenerated

[2] This equation is identical to the solution obtained by Tahvonen and Salo's utility maximisation model when assuming perfect capital markets and excluding amenity values (Tahvonen and Salo, 1999).

areas have no amenity values, $\pi(0) = 0$. The solution of the first-order linear differential equation given by (11) integrating by parts yields

$$\lambda_2(\tau_j^+) = \int_{\tau_j^+}^{\tau_{j+1}^-} \pi'(t - \tau_j^+) e^{-\delta t} dt =$$

$$e^{-\delta t} \pi(t - \tau_j^+)|_{\tau_j^+}^{\tau_{j+1}^-} + \delta \int_{\tau_j^+}^{\tau_{j+1}^-} \pi(t - \tau_j^+) e^{-\delta t} dt =$$

$$\pi(\tau_{j+1}^- - \tau_j^+) e^{-\delta \tau_{j+1}^-} - \pi(0) e^{-\delta \tau_j^+} + \delta \int_{\tau_j^+}^{\tau_{j+1}^-} \pi(t - \tau_j^+) e^{-\delta t} dt \quad (20)$$

Now the cutting rule, applying equation (15), (17) and (20), is expressed as

$$pe^{-\delta \tau_{j+1}^-} F(x(\tau_{j+1}^-)) + \pi(\tau_{j+1}^- - \tau_j^+) e^{-\delta \tau_{j+1}^-} + \delta \int_{\tau_j^+}^{\tau_{j+1}^-} \pi(t - \tau_j^+) e^{-\delta t} dt$$
$$- \pi(\tau_j^- - \tau_{j-1}^+) e^{-\delta \tau_j^-} - pe^{-\delta \tau_j^-} F(x(\tau_j^-)) =$$
$$- \delta [px(\tau_j^-) - c_p x_p] e^{-\delta \tau_j} \quad (21)$$

This rule gives the optimal rotation length which maximizes the timber and non-timber benefits. The Hartman rule can be obtained from this cutting condition if it is expressed in rotation intervals, T_i. Considering identical infinite rotations the Hartman equation have the following form,

$$pF[x(T_i)] + \pi[x(T_i)] = \delta px(T_i) + \frac{\delta[px(T_i)e^{-\delta T_i} - c_p x_p]}{1 - e^{-\delta T_i}} + \frac{\delta \int_0^{T_i} \pi[x(t)] e^{-\delta T_i} dt}{1 - e^{-\delta T_i}} \quad (22)$$

This equation is identical to the Faustmann equation but it includes the additional flow of amenity outputs if the harvest is delayed and the 'site value' includes both timber and non-timber benefits.

3 Conclusions

This paper has proposed a bioeconomic framework for the analysis of rotational dynamics in forest management. This is done by modelling harvest as jump controls. The paper shows that the Faustmann model is a special case of a much more general class of models. The Faustmann and Hartman rules emerge when the cyclical dynamics of forest management are constrained to clear-cutting. This emphasises that the optimal control framework including jump controls constitutes a flexible modelling alternative to the Faustmann tradition. Future research based on this framework will be able to model a broader range of forest regimes which capture the diversity of biological and economic aspects inherent in forest ecosystem management.

References

Anderson, F. J. (1976). Control theory and the optimum timber rotation. *Forest Science 22(3)*, 242–246.

Bentley, W. R. and D. E. Teeguarden (1965). Financial maturity: A theoretical review. *Forest Science 11*, 76–88.

Chang, S. J. (1982). An economic analysis of forest taxation's impact on optimal rotation age. *Land Economics 58(3)*, 310–323.

Chang, S. J. (1983). Rotation age, management intensity, and the economic factors of timber production: Do changes in stumpage price, interest rate, regeneration cost, and forest taxation matter? *Forest Science 29(2)*, 267–277.

Clark, C. W. (1990). *Mathematical Bioeconomics: The optimal management of renewable resource* (Second ed.). New York: Wiley.

Englin, J. E. and M. S. Klan (1990). Optimal taxation: Timber and externalities. *Journal of Environmental Economics and Management 18*, 263–275.

Faustmann, M. (1849). On the determination of the value which forest land and inmature stands possess for forestry. *English edition edited by M. Gane, Oxford Institute Paper 42, 1968. Reprinted in Journal of Forest Economics (1995) 1(1):7-44*.

Hartman, R. (1976). The harvesting decision when a standing forest has value. *Economic Inquiry 14*, 52–58.

Hof, J. (1993). *Coactive Forest Management*. London: Academic Press.

Johanson, P.-O. and K.-G. Löfgren (1985). *The Economics of Forestry and Natural Resources*. Oxford: Basil Blackwell.

Koskela, E. (1989). Forest taxation and timbeer supply under price uncertainty: Credit rationing in capital markets. *Forest Science 35(1)*, 160–172.

Kuuluvainen, J. (1990). Virtual price approach to short-term timber supply under credit rationing. *Journal of Environmental Economics and Management 19*, 109–126.

Löfgren, K.-G. (1983). The faustmann-ohlin theorem: A historical note. *Hystory of Political Economy 15(2)*, 261–264.

Max, W. and D. E. Lehman (1988). A behavioral model of timber supply. *Journal of Environmental Economics and Management 15*, 71–86.

McConnell, K. E., J. N. Daberkow, and I. W. Hardie (1983). Planning timber production with evolving prices and costs. *Land Economics 59(3)*, 292–299.

Montgomery, C. A. and D. M. Adams (1995). Optimal timber management policies. *In Bromley D. W. (ed.) Handbook of environmental economics*, 3–14.

Naslund, B. (1969). Optimal rotation and thinning. *Forest Science 15*, 446–451.

Newman, D. H., C. B. Gilbert, and W. Hyde (1985). The optimal forest rotation with evolving prices. *Land Economics 61(4)*, 347–353.

Ohlin, B. (1921). Till frägan om skogernes omloppdtid. *Ekonomisk Tidskrift, 22. Reprinted in Journal of Forest Economics (1995) 1(1)*, 89–144.

Paredes, G. L. and D. J. Brodie (1989). Land value and the linkage between stand and forest level analysis. *Land Economics 65(2)*, 158–166.

Pressler, M. R. (1860). Aus der holzzuwachslehre. *Allgemeine Forst- und Jagdzeitung, 36. Reprinted in Journal of Forest Economics (1995) 1(1):45-87.*

Reed, W. J. (1984). The effects of the risk of fire on the optimal rotation of a forest. *Journal of Environmental Economics and Management 11*, 180–190.

Samuelson, P. A. (1976). Economics of forestry in an evolving society. *Economic Inquiry 14*, 466–492.

Schreuder, G. (1971). The simultaneous determination of optimal thinning schedule and rotation for an even-aged forest. *Forest Science 17*, 333–339.

Seierstad, A. and K. Sydsaeter (1987). *Optimal Control Theory with Economic Applications*. Amsterdam: North-Holland.

Snyder, D. L. and R. N. Bhattacharyya (1990). A more general dynamic economic model of the optimal rotation of multiple-use forest. *Journal of Environmental Economics and Management 18*, 168–175.

Strang, W. J. (1983). On the optimal forest harvesting decision. *Economic Inquiry 4*, 576–583.

Swallow, S. K., P. Talukdar, and D. N. Wear (1997). Spatial and temporal specialization in forest ecosystem management under sole ownership. *American Journal of Agricultural Economics 79*, 311–326.

Tahvonen, O. (1998). Bequests, credit rationing and in situ values in the faustmann-pressler-ohlin forestry model. *Scandinavian Journal of Economics 100(4)*, 781–800.

Tahvonen, O. and S. Salo (1999). Optimal forest rotation with *in Situ* preferences. *Journal of Environmental Economics and Management 37*, 106–128.

Termansen, M. (2000). The optimality of rotational dynamics in forestry. *Paper presented to the tenth annual conference of the European Association of Environmental and Resource Economists, Rethymnon, Crete, June 30-July 2*.

Wacker, H. (1999). Optimal harvesting of mutualistic ecological systems. *Resource and Energy Economics 21*, 89–102.

Willassen, Y. (1998). The stochastic rotation problem: A generalization of faustmann's formula to stochastic forest growth. *Journal of economic Dynamics and Control 22*, 573–596.